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able to jump any height up to five feet. However high he may jump, he could always have gone a little higher (which is at least as easy to admit as the contrary). Hence he could jump any height. But in reality, $x_0 = 5$ and $(5, f(5), ff(5), \cdots)$ is a *convergent* sequence whose limit is probably less than 7.

RECENT PUBLICATIONS.

REVIEWS.

Dynamic Symmetry, the Greek Vase. By JAY HAMBIDGE. New Haven, Yale University Press, 1920. 161 pages, small folio. Price $6.00.

A mild sensation has been caused among classicists, artists and the philosophers of æsthetics by the doctrine of "dynamic symmetry" as recently enunciated and promulgated by Mr. Jay Hambidge and his converts. Certain ambitious claims have been made which have been greeted, possibly with scepticism and repugnance by some artists and classicists, but certainly with interest and sympathy by many readers, including the Yale School of Fine Arts, and the members of various museum staffs. The exposition has been clothed in a certain amount of numerical terminology, so that many who would, perhaps, desire to approach the subject with an open mind may have been discouraged at the start and led to regard the matter as too technical for the comprehension of any one not deeply versed in the intricacies of higher mathematics.

So far as the reviewer is aware, this work has not received from professional mathematicians the attention that it deserves, particularly since any adequate criticism of the results claimed can come only from one prepared properly to appraise the mathematical background of the argument. The book under review is assuredly not intended as a reference work in geometrical analysis, but in its wider philosophical aspects, its claim for the historical and logical application of geometry to design must merit the consideration of any one who is interested in the place of mathematical discipline among the cultural developments of civilization.

Let us first cite some passages in which Mr. Hambidge enunciates his claims. "Some twenty years ago, the writer, being impressed by the incoherence of modern design and convinced that there must exist in nature some correlating principle which could give artists a control of areas, undertook a comparative study of the bases of all design, both in nature and in art. This labor resulted in the determination of two types of symmetry or proportion, one of which possessed qualities of activity, the other of passivity. For convenience, the active type was termed dynamic symmetry, the other static symmetry. It was found that the passive was the type that was employed most naturally by artists, either consciously or unconsciously; in fact no design which could be recognized as such—unless indeed it were dynamic—would be possible without the use, in some degree, of this passive or static type. . . . A study of the basis of design in art shows that this active symmetry was known to but two peoples, the Egyptians and the Greeks, the latter only having developed its full possibilities for purposes of art. The writer believes that he has now recovered, through study of natural form and shapes in Greek and Egyptian art, this principle for the proportioning of areas. . . . At some time during the Sixth or Seventh Century B. C. the Greeks obtained from Egypt knowledge of this manner of correlating elements of design. In their hands it was highly perfected as a practical geometry, and for about three hundred years it provided the basic principle of design for what the writer considers as the finest art of the Classic period. Euclidean geometry gives us the Greek development of the idea in pure mathematics; but the secret of its artistic application completely disappeared. Its recovery has given us dynamic symmetry—a method of establishing the relationship of areas in design composition. . . . This idea (the application of areas) is quite unknown to modern art but that it is of the utmost importance will be shown in this book by the analysis of Greek vases. . . . Analysis of Greek and Egyptian composition shows that the artist always worked within predetermined areas. The enclosing rectangle was considered the factor which controlled and determined the units of the form. . . . Analysis of any fine Greek design is sure to disclose an arrangement of area which produces the quality of inevitableness, so conspicuously absent in modern art."

Mr. Hambidge when speaking of mathematical notions does not employ in all particulars customary mathematical language. This may be due to a desire to escape from the unnecessarily technical, but in any case it makes the reading more difficult for the mathematician and probably meaningless for the average artist. When he speaks of "proportioning areas," it has no reference to the familiar artistic principle often discussed under the term, space balance, or the balance of areas, which will be referred to later. The "proportioning of areas" means here, in effect, the determination of significant points and horizontal lines within a rectangular diagram by use of geometrical methods.

A condensed summary of the claims made for dynamic symmetry as applied to the Greek vase may be expressed in the following two statements concerning the profile curve of the lateral elevation of the vase, including the handles if such there be. On the negative side—except for the fundamental bilateral symmetry of the design, equipartition of ordinates and of abscissas is avoided, and any regularity of linear division that may exist avoids simple commensurate ratios. On the positive side—while commensurate ratios are avoided, the incommensurate ratios occurring are such that their squares or rational combinations among these squares are simply commensurable. This means incidentally, as every mathematician knows, that the significant points and lines can be located by an elementary use of ruler and compasses.

To what extent has Mr. Hambidge established his theory? As in the case of other enthusiasts there is among his remarks no hint of doubt or failure. He is convinced whole-heartedly of his claims and expects the reader to acknowledge as ample the evidence displayed. Analysis is given in drawings or tables of many examples and types. A count of these scattered throughout the text shows the following distinct specimens, here collected and listed by type: amphora (15), small cup (1), dinos and stand (1), hydria (3), kalpis (7), kantharos (2), bell krater (3), volute krater (1), kylix (19), lekythos (10), olpe (3), oinochoe (2), pelike (3), perfume vase (1), psykter (1), pyxis (3), skyphos (14), stamnos (2), a total of 91 cases analyzed. While this is of course an inconsiderable proportion of the extant Greek vases, yet results found to hold uniformly in the examination of the first 91 random samples may be fairly assumed to establish the presumption that the thesis is valid throughout.

In emphasizing the beauties of his new theory, Mr. Hambidge appears to have overstressed the lack of principle in current theories of design. The inevitableness and geometrical background that he claims as the characteristic features of the doctrine of dynamic symmetry have not been rediscovered and expounded here for the first time. Let me quote from E. A. Batchelder's conservative *Principles of Design:* "A finished and satisfactory design should never suggest the question, 'Can I remove this feature'? or 'Is that line necessary'? There should be no question about the importance of each line involved" (page 25). Again, "The law of balance in so far as we have gone may be concisely stated as follows: 'Equal measures of equal contrasts will balance at a point midway on a line connecting the centers of the measures. Unequal measures of equal contrasts will balance on a line connecting the centers of the measures at distances which will be in inverse ratio to the measures'" (page 42). "The Indian has not yet acquired sufficient grasp on the principles of his art to allow his imagination to express itself unrestricted by the limitations of geometry. Japanese art, on the other hand, to the casual observer, contains very little of geometry; but the student of the art of Japan knows that the work of that country rests just as firmly upon the fundamental principles of design as the work of any other nation or period" (page 44).

Mr. Hambidge seeks to forestall certain obvious questions. Most of his measurements are made by the members of the staffs of the museums which contain the vases analyzed, and he takes care to remark in some cases that the drawings were not made to fit the measurements expected, but were first executed carefully to scale, and subsequently analyzed. A not unnatural basis for scepticism he disposes of as follows: "The shape of the vessel is determined with great care" and "shows unmistakably that the picture (the red-figured decoration upon the vase) is secondary. . . . The idea that so much care should be taken to proportion such a commonplace article as a clay pot will probably strike the average reader as fanciful. And it would be so if ordinary pottery were under consideration. The vases considered here, however, are Greek, and the Greek vase is unique. . . . Volumes have been written about the pictures found on Greek pottery, but the shape or form of the vase, which is of much greater importance, has been almost entirely neglected. . . . To the ancient Greek, the form of the vase was of vital importance, the vase painting was usually of secondary importance, a fact made clear by the great preponderance of signatures of potters over those of painters."

Mr. Hambidge has assuredly succeeded in exhibiting certain vase forms in which the simple mathematical proportions of the parts are too convincing to be dismissed as accidental. In particular his single example of a small cup shows each of its important points identified by the simplest sort of construction. The fact that many of the vases are "exactly" inscribed in a square, that so often among the examples discussed the width of the foot is "identical" with that of the lip seems hardly likely to be a mere coincidence. However the reviewer at least is far from being convinced as to the general claim.

The question reduces as to whether evidence has been shown that Greek vases were regularly designed with mathematical accuracy in all essential parts to accord with measurements in a diagram constructible with ruler and compasses. The independent incommensurables referred to by Mr. Hambidge are, in particular, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, the so-called "whirling square" ratio being the positive root of the equation, $1/(x - 1) = x$, namely, $\frac{1}{2}(1 + \sqrt{5})$.

I shall first point out what I regard as fatal flaws in the argument for the historical significance of dynamic symmetry. To put the matter briefly, it seems that the reader is at liberty to question both the exactness and the adequacy of the analyses submitted in the book.

The impression is conveyed that most measurements displayed check to a remarkable degree of exactitude with the drawings exhibited. This impression is heightened not only by explicit statements to that effect but also by the careful pointing out and explanation of a few noticeable but not serious departures. Occasional remarks however have slipped into the text that invite a contemplative pause. Thus, "The percentage of error is much smaller in the bronzes than in the pottery." The term "error" is clearly used to denote discrepancy between assumed analysis and measured dimensions. It is enlightening that no hint of recurrent percentage error in the pottery is made until it is desired to emphasize the accord of the bronzes with assumed canons. A detailed explanation is offered as to why the handles of the kylikes should show such random heights in their relations to the bowls. The explanation sounds more ingenious than plausible, but in any case admits an irregularity as to heights that is thereafter ignored. One wonders why out of a list of fifty-four analyzed kylikes, a table of but seventeen is given. One suspects, perhaps unfairly, that the remaining examples fail to bear out the theory in so obvious a manner. Throughout the treatise the reviewer feels that there is a seeming inclination to overlook or condone apparent discrepancies, and little suggestion of an impartial statistical attitude that would invite and compel conviction.

The adequacy of the analysis may be criticized on many counts. The vase profiles are in general interesting curves not identified by means of a few points and tangents. One is led at once to inquire whether the analysis covers the determination of these most important features of the design. The author says, "No mathematical curves have, so far, been found in Greek art. . . . Curves were apparently drawn by tangents in this manner (with assigned tangents at certain points) all through the Greek classical period." Certain portions in a design which do not lend themselves to convenient analysis are referred to as "loosely rendered." A cursory survey of the analyses submitted is pleasantly satisfying, but a closer inspection frequently reveals what seems little more than random selection. Certain conspicuous points in the design are located in the system, but others quite as important are not included among those present and accounted for. It is particularly striking that of two vases practically identical to the eye, the analysis should in one case include certain points of the vase profile, while for the other a set notably distinct should be chosen for special mention. One feels instinctively that the featured points were signalized *a posteriori* as being those which appeared to fit passably into an inflexible and inadequate mould. In several cases the drawings exhibit the actual asymmetry of the material vase, and lines determining intersections for the right half of the vase would fail for the left, leaving the actual significance of the points plotted entirely in doubt.

But there are much stronger reasons for challenging the principal argument of the book. One may be.prepared to agree with Mr. Hambidge in believing that Greek architecture and architectural design employed dimensions accurately laid out by ruler and compasses in incommensurable as well as in commensurable ratios. An undertaking in which the designer and the artisan are distinct, such as building a temple, gave ample scope for the use of working drawings, laid out carefully by such geometrical drafting instruments as were available. Temples were not essentially commercial and were not multiplied daily. There is however ample evidence that the Greek vases, except only for some of very large dimensions, were turned on a potter's wheel; that the designer was in many of the finest examples also the executor, and could therefore judge with his eye as to esthetic excellence as easily as measure the wet clay against a scaled drawing. That no two Greek vases are known of identical form, despite the rigid commercial types into which they fell, is almost proof positive that no exact measurements were intended. Vases were manufactured by the hundreds and thousands and for strictly utilitarian ends. Their beauty and their charm were merely reflections of the maker's pride in careful and honest workmanship. All authorities agree that the finest Greek vases were never intended as mere decorations. Their design as well as their execution was in the hands of the metics, a skilled but usually poorly educated class of artisans. It is next to inconceivable to me that a complicated working drawing was made afresh for each vase, that differences scarcely noticeable to the eye were purposely secured at great labor by extensive and thoroughgoing alterations in the pattern of construction lines. Yet this is what Mr. Hambidge would have us believe. Granted that the curves themselves were not constructed by mathematical methods, what right have we to suppose that the critical points of the design need have been so chosen?

It remains to dispose of the immediately pertinent inquiry as to how one can account for the remarkable analyses submitted by Mr. Hambidge. My answer is a simple one, perhaps too simple to appeal to the fancies of any who have caught that distinguished illustrator's enthusiasm for dynamic symmetry. The author uses the square roots of 2, 3, and 5, and all that can be obtained from these as linear expressions with rational coefficients. He reserves also the privilege of utilizing other quadratic surds in the same manner but appears to find these sufficient. Among the infinite set of numbers between zero and unity that can be so obtained, I have noted 53 distinct ones which the author mentions explicitly in the course of the text. Two of these, namely, $2\sqrt{5} - 4$, which he writes as 472, and $\sqrt{2/3}$ which he writes as .4714 (the text employs decimal notation throughout), differ by approximately one part in fourteen hundred. Thus by using this simple difference alone we can obtain something like fourteen hundred intervals between zero and unity. The reply to the query is then this obvious one, namely, that Mr. Hambidge has provided himself with so bountiful a list of incommensurables that any observed measurement may be expressed by the aid of these to a degree of precision beyond the most fastidious ambitions of the potter. There is a single table in the book exhibiting certain relative dimensions of several vases. Without being here concerned with the particular significance of the data selected, we shall write down for comparison two possible sets. The first row is selected from corresponding entries in the table which are supposed to illustrate, by their numerical form, "root two figures." These are to be contrasted with those of the second row which has been selected as illustrating figures which are not "root two" in theme. The numbers have been chosen from the columns given by the author, although it is but fair to remark that the numbers here given do not all happen to occur in a common row in the book. These rows are as follows:

3.4142, 2.4714, .9393, .2929, .7071, .3535;

and

3.382, 2.472, .927, .309, .691, .354.

These dimensions would be regarded by the author as characteristic of vases of markedly different type. It is beyond the present credibility of the reviewer to conceive as impossible that the analyst in measuring two vases, one with the ratios of the first row, and the other with the ratios of the second row, might not utilize his assumption of percentage errors and fall into the "confusion" of mistaking these two shapes as negligible variants from a common design. Mr. Hambidge even remarks in one connection that the analysis may not be unique, although perhaps he there refers to the geometrical steps followed and not to the numerical data assigned.

Unlike most books on design, this text explicitly warns the inquiring artist against applying purely esthetic criteria, informing him that to emulate Greek perfection, it is essential and apparently sufficient to acquire facility in manipulating the elementary arithmetical incommensurables. The author does not hesitate to praise one vase above another, both of which are supposed to comply equally with the canons of numerical exactitude. How to design a *good* vase as distinguished from an exact but ill-conceived one is not discussed. And is not this the most important question suggested by the whole discussion?

For its probable future influence, its suggestive analyses, its rich lore of illuminating and interesting mathematical and archeological references, and not least for its exquisite half-tones of rarely charming vases, this book is recommended for the attention and study but not the blind acceptance of such as appreciate science and art.

ALBERT A. BENNETT.

Calculus and Graphs—Simplified for a First Brief Course. By L. M. PASSANO. New York, The Macmillan Company, 1921. 12mo. 8+167 pp. Price $1.75.

As every one is aware, who has had the pleasure of teaching the subject of calculus, many features of the subject present no essential difficulties even to students who have shown no special aptitude along mathematical lines. The mere technique of differentiation can be readily acquired by students who cannot learn facility in the manipulation of trigonometric identities. The chief difficulty in solving elementary problems seems to lie in the lack of ability displayed by the student in converting a verbally formulated problem into tractable equations. This is a difficulty that is earlier presented by the subject of algebra, and has nothing to do with any one particular branch of mathematics. As long as verbal problems are offered for analysis, there will be students who find the subject difficult. In the present book there has been no effort to spare the student in this direction. Since the development of clear reasoning is one of the primary purposes in any mathematical course, this book assuredly fulfils its principal aim.

Of the other difficulties which the subject sometimes presents may be mentioned the extensiveness of the field of applications, and questions of rigor connected with the notion of limit. In the text under discussion, the material is wisely restricted so as to make possible an emphasis upon essentials without which the student can receive no benefit from the course. The author gives us the following warning in his preface: "The aim has been to make the student understand the subject; not to write a book that would satisfy meticulous mathematical pedantry. In so doing the author may have in places sacrificed logical detail to simplicity of presentation, but never, he hopes, accuracy of statement. In the opinion of the writer a too rigidly logical proof with its paraphernalia of subscripted Greek letters is out of place in an elementary first course in calculus, for the reason that the student never understands such a proof. Or if by arduous effort he does grasp its meaning, it is at the expense—in time and labor—of other things that are more important and far more useful." In view of this announcement the reader need not be surprised to see infinite series used where necessary without a word about convergence; but some other features are not so readily explained. On page 15, we read, "We do not know whether $x + 2$ is plus or minus zero; that is, whether $x + 2$ is an exceedingly small positive or an exceedingly small negative number." On page 19, and repeatedly throughout the book, "The increment of the function can be made smaller than any number we may assign, however small, by making the increment of the independent variable small enough . . .," where the word "number" is apparently expected to mean